THE BELIEF OF MATHEMATICS LEARNING

Comparative study of textbooks and case studies of classroom

Focusing on the application of figurative representation

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CONTENTS

THE PURPOSE

- **This study investigates the cultural belief about mathematics learning.**
  - In school, pupils learn almost the same topic or idea, but the order or the grade when pupils learn and the detail material is a little different.
  - And teaching method may have similarity, but also have difference.
- Well, the belief about what aspect of mathematics pupils should learn is the same or different?
  - What aspect is the same ? What aspect is different?
  - This is the big question of my study. .

TEACHING AS A CULTURAL ACTIVITY

- Comparative Video Studies of learning process
  - "The Teaching Gap" (Stigler and Hiebert, 1999)
  - Santagata (2004, 2005)
    - Focusing on teachers' mistake handling
    - Compare between Italy and America
- **The object of the Comparative analysis in this study is textbooks, which provide teaching.**
  - examine meta-discourse of textbooks.
  - conduct verification of Japanese cases

FOCUS IN THIS STUDY

Diagrams and Problem-solving

- Diagrams are effective for learning mathematics.
  - This study applies diagrams as figurative representation.
  - The role of Modeling and representing
  - Tools for understanding and problem-solving.
- Problem-solving is a common activity in mathematics learning.
- **In Japan, teachers apply the diagrams as the traditional math teaching method.**
  - Japanese community of Mathematics education invented and developed them. And authorized textbooks adopt the fundamental concepts of them, and arranged them to make it easy for learners to use.
- **And they have elaborated "problem-solving learning" by reference to several theories.**

METHOD

- **Comparative study about Meta-discourse of textbooks**
  - Meta-discourse of textbooks may inform the cultural belief because the textbooks are edited by teachers and educators of each country.
  - The feature of Japan may emerge from this study.
- **Case studies of Japanese math class**
  - Case studies may clarify how to learn mathematics through problem-solving and the use of diagrams and what teachers have pupils learn.
- **What school math aim at**?

COMPARATIVE STUDY OF TEXTBOOKS

China, Singapore and Japan

What is the difference of the belief about school mathematics

Method of Textbook-analysis

- Comparative study; China, Singapore and Japan
  - Two publishers per each country
  - Contents; multiplication and division of decimals
  - For 4th and 5th graders
- Approach; similarity and difference
  - Compositions and layouts
  - Quantitative approach
    - Samples and Exercises
    - Word problems with/without situation.
  - Qualitative approach
    - Feature Layouts
    - Kinds of diagrams

Textbook-analysis

CHARACTERISTICS

Textbook-analysis

China

- A few samples and many exercises
- Word problems always have everyday situations.

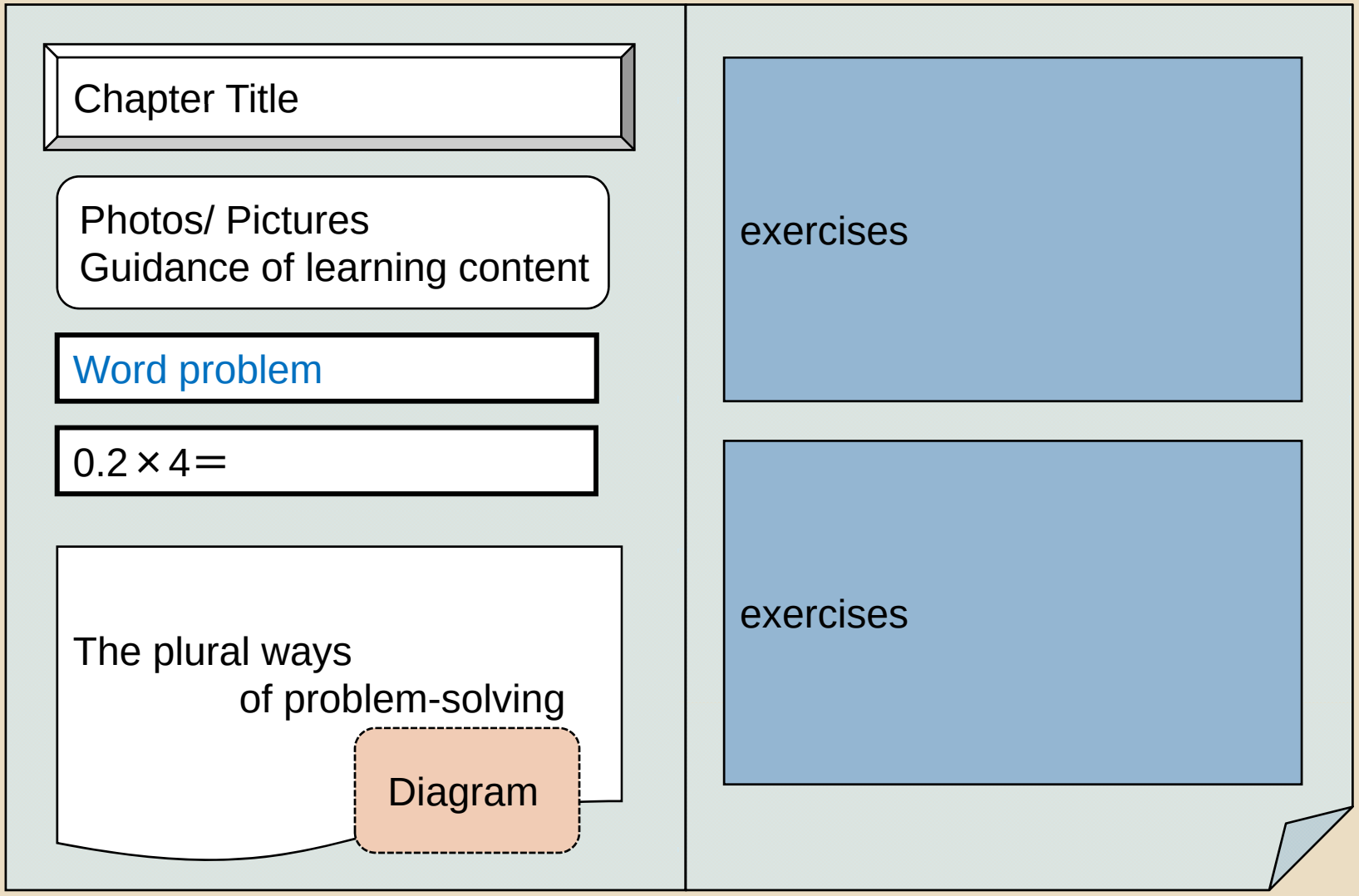
Singapore

- more questions; More samples and fewer exercise
- Word problems do not always have situations.

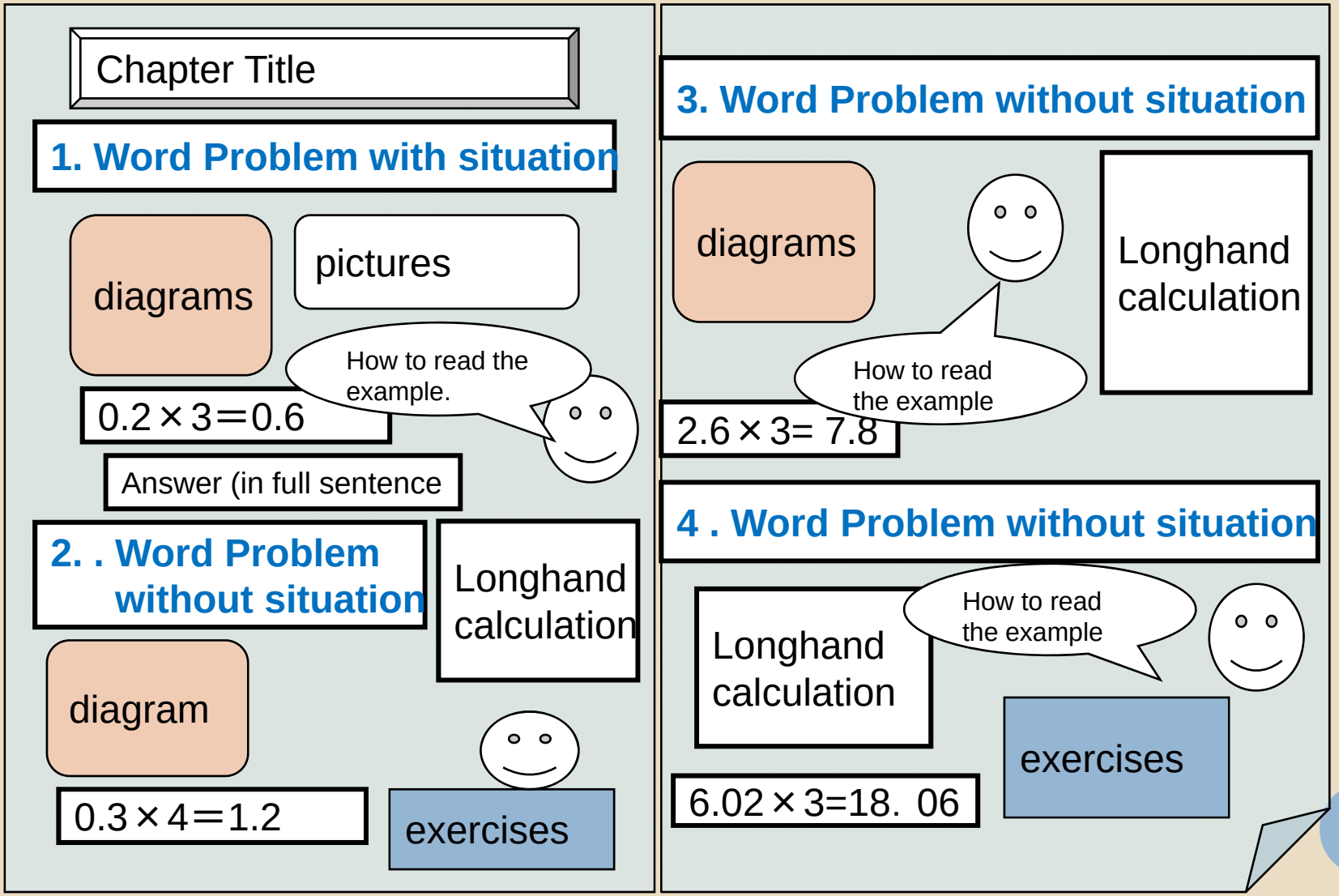
Japan

- A few questions ; exercises are as many as samples.
- Most of Word problems have situations.

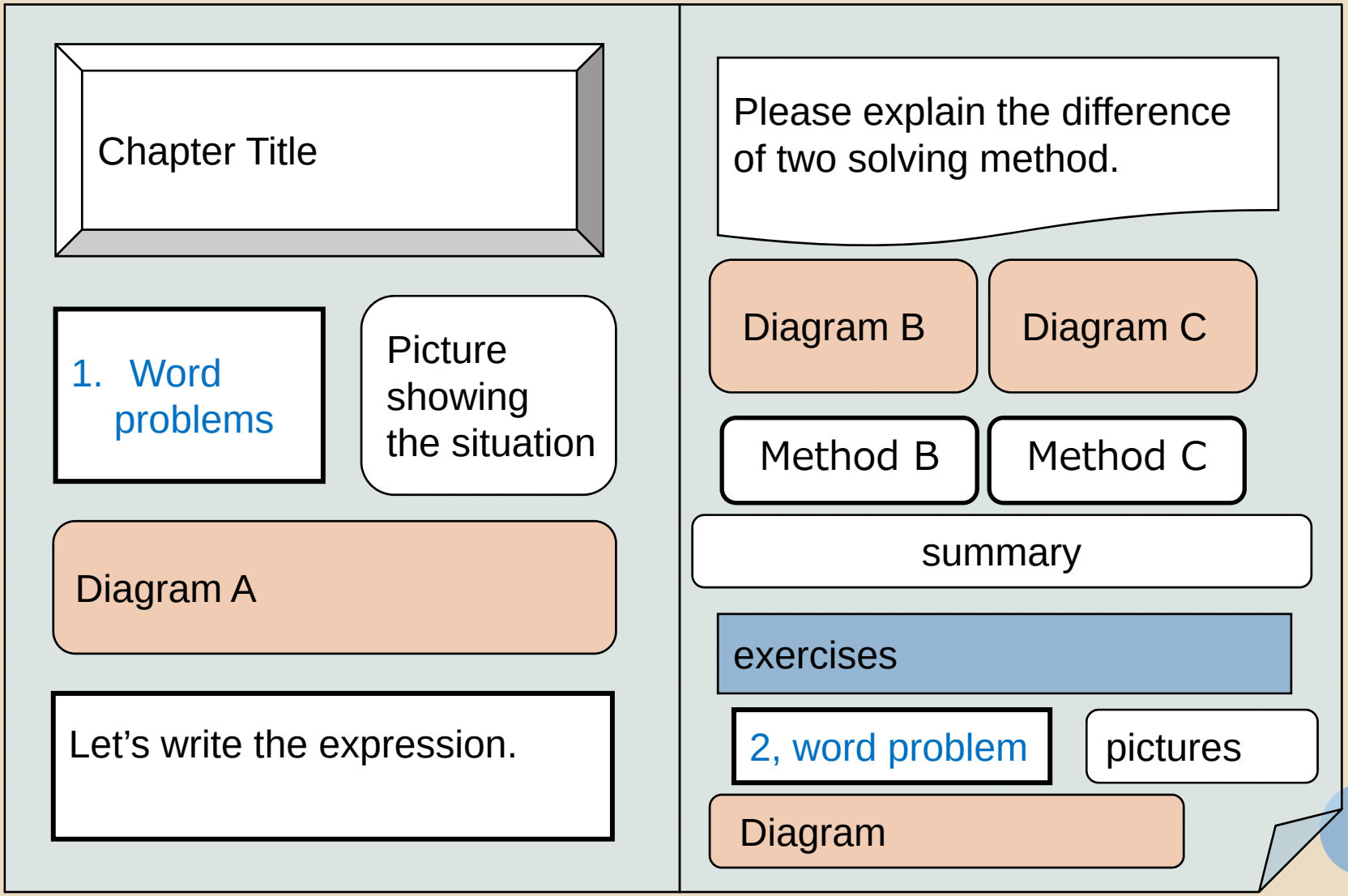
Feature Layout ; China



Feature Layout ; Singapore

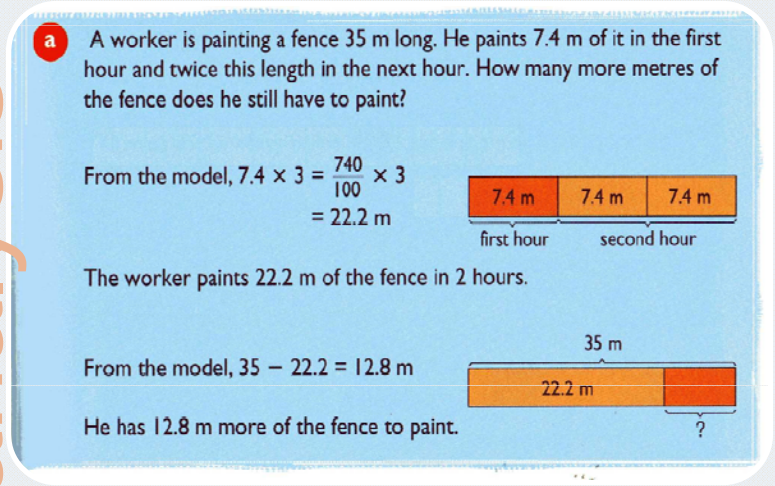


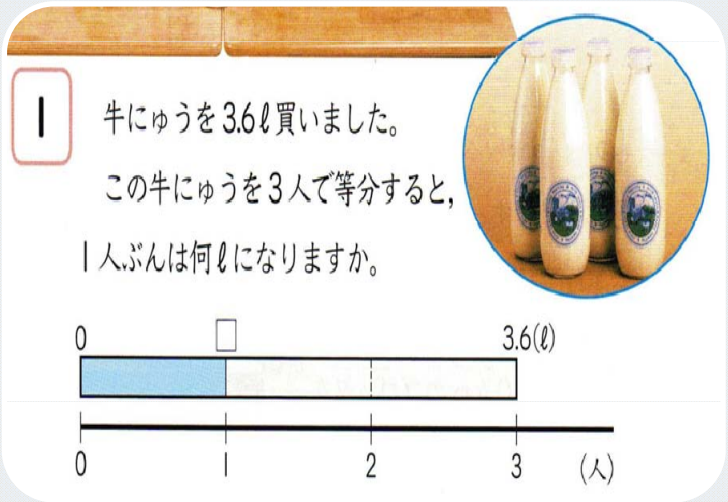
FEATURE LAYOUT ; JAPAN



Examples of diagrams; Singapore and Japan

Textbook-analysis





Singapore

- Texts-specific model
- Simple model
- Drawing helps PS.

Japan

- Domain-specific model
- Complex model
- Reading helps PS.

DISCUSSION; BELIEF OF SCHOOL MATHEMATICS

Textbook-analysis

China **=knowledge-centered**

- Apply the knowledge to PS
- Diagrams are the knowledge

Singapore **=strategy-centered**

- Acquire the PS skill through PS tasks
- Diagrams are one of the strategies for PS

Japan **= Activity-centered**

- Do exploring through PS activity
- Diagrams are tools for thinking and explanation to others

ACTIVITY-CENTERED?

- What/How they learn through activity?
- What aspect of learning they consider important through PS?
- What aspect of PS the diagrams help?

⇒Next phase of this study is case studies of Japanese class to examine the belief of Japanese school mathematics more clearly.

Case studies Learning Process in Japan

CASES

- Case 1
  - 6th graders
  - division of fraction
  - Teacher's intention
    - To avoid just numeral explanation because he think of it as immature one.
    - To use the diagrams is helpful for their learning more complicated contents later. Ex) division of fractions
- Case 2
  - 5th graders
  - multiplication of decimal
  - Teacher's intention
    - To understand both solving ways in textbooks.
    - And to explain what is different to other pupils who can not understand enough.

Point of View

- The process of problem-solving
  - What they talk about?
  - Why it is necessary to talk about?
- The roles of diagrams
  - What kind of diagrams they used ?
  - What they help pupils to do?

1 We used 2/3 ℓ of fertilizer. How many litters need per 1㎡?

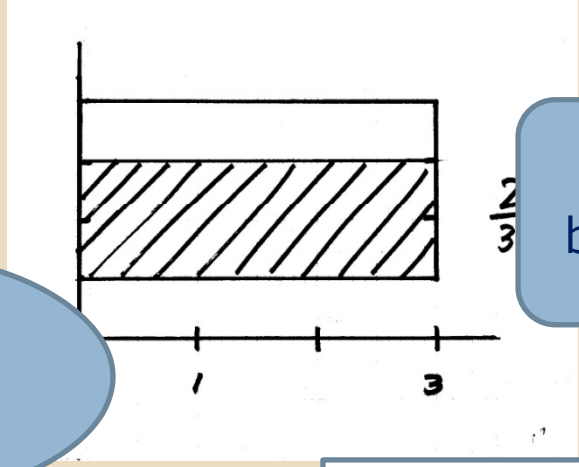
$$\frac{2}{3}\div 3$$

$$=\frac{2}{3}\div\frac{3}{1}=\frac{2}{3}\times\frac{1}{3}=\frac{2}{9}$$

$$=\frac{2}{3}\div\frac{3}{1}=\frac{2\div 3\times 3}{3(\div 1)\times 3}=\frac{2}{9}$$

Most pupils explained the solving way **by the operation of the expression**. And then**, the teacher asked them to explain the meaning** of 3×3 **with the diagram.**

They began to solve another problem.



Pupils tired to use the diagram.

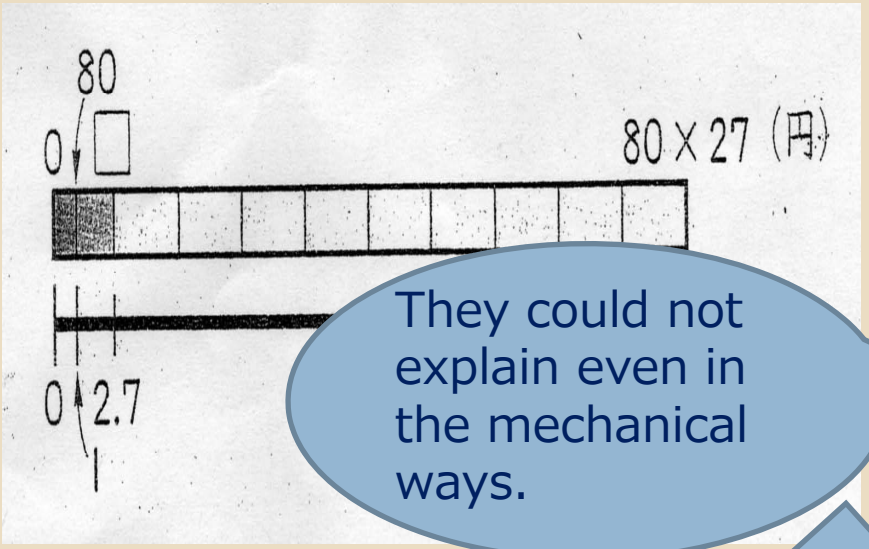
Teacher found they shared the diagram, but they did not share the meaning of it.

Kino: 1 was divided into 9.
T: What is 1?
C:1 ℓ ! /1㎡!
T:which is correct?

Some students didn't understand the diagram enough.
So ,it cause the question, "where 1 is?"

After discussion, They could share the understanding of the diagram.
This led them to the understanding of the first Kino's explanation.

2 The price of the ribbon is 80 yen per 1m. If you buy 2.7m, how much does it cost?



80 ×2.7
(method 1)
=80÷10×27
⇒ 80÷10…
⇒ 27…10tim
(method 2)
=80×27÷10
⇒ 80×27… 2160yen /27m
⇒ ÷10… 2.7 =1/10 of 27m

They seemed to understand the diagram in the session about method 1.

They could not explain even in the mechanical ways.

No pupils could explain method 2 enough. **And then, the teacher told them to use the diagram for their understanding.**

(in Small Group)
Ama seemed to be
at a loss because he didn't
diagram. But Ama could say
nothing.

Although a boy could find the answer, they could not explain the procedure with diagram.

(w
So
th
tea
ca
Du
gra
referring to the diagram.

To use the diagram for mediating tools enable him to understand the meaning of diagram and caused re-understanding.
So, the diagram is also used as a collective thinking tool.

RESULT

- **Their "problem" is not just "the word problem"**
  - They discussed the meaning of the diagram.
  - Solving the word problem was easy, but the explanation of why the answer was correct was difficult.
  - So, through problem-solving, they articulated their understanding.
- **It is more important to understand the process of calculation than to calculate correctly.**
  - To share the meanings of diagram is to have common expressions in the class.
  - So , the use of them lead to mathematical communication and constructive understanding.
- **The diagrams play the role of the mediation.**
  - They mediates everyday expression and mathematics expression.
  - Moreover , they mediate participants.

Textbook-analysis

THE CHARACTERISTICS OF JAPANESE BELIEF

- **Activity-centered learning supports** pupils' learn to **think and communicate mathematically** through collective problem solving.
  - In comparing the plural ways of problem-solving and thinking.
  - In a sample word problem with some questions.
    - Explanation of the procedure and thinking way.
    - Application of what they have learned
  - Let's explain to others who can not understand it.
- **Understanding Complex diagram support s mathematical thinking and communication**
  - It is necessary to understand the diagrams themselves for applying the diagrams.
    - Indeed, it is difficult, but understanding them leads to problem-solving and understanding deeply because they originally relate to the concept understanding.

CONCLUSION

- The application of diagrams plays different roles in each country/culture and stems from the cultural belief.
- And the belief relates to the competence that they want pupils to acquire.
- They need to become aware of the learning possibility of other competences through the same activity.
  - Some Japanese teachers says the diagrams are difficult to use because pupils could not use the diagram as they expected. But if pupils don't understand the diagram, it is natural that they should not use effectively. So, teachers need to understand the purpose of the application of diagram.

Toward Future Studies

- Comparative studies of combination approach to textbooks and learning processes.
- More investigation with other countries.
- These studies will show what/ how they should learn in school mathematics along their culture and social background.

THANK YOU

謝謝

A-RI-GA-TO-U

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Table1 1 : textbooks

Country	publishers	grade	page
China	J	4th	1-38
	B	4th	38-78
Singapore	M	4th	52-73
	I	4th	68-88
	P	5th	8-36
	S	5th	2-29
Japan	T	5th	19-33, 73-98
	K	5th	41-53, 68-88

Table 2 ; samples and exercise

	Pub.	pages	examples *per page*	exercises *per page*	WP With sit	WP Without sit.
China	J	39	15 *0.38*	54 *1.38*	15	0
	B	41	11 *0.27*	31 *0.75*	17	0
Singapore	M	21	34 *1.62*	19 *0.94*	17	24
	I	23	30 *1.30*	69 *3.00*	36	31
	P	25	28 *1.12*	26 *1.04*	20	0
	S	28	34 *1.21*	23 *0.82*	27	33
	M+P	46	62 *1.35*	45 *0.98*	37	24
	I+S	51	64 *1.25*	92 *1.80*	63	64
Japan	K	34	29 *0.85*	35 *0.83*	33	2
	T	41	29 *0.71*	35 *0.85*	42	4

WP:Word Problems / Sit. ; Situation

Example; the questions with the answer

Exercise ; the questions after examples

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