The Learning & Teaching of Equivalent Fractions for Conceptual Understanding

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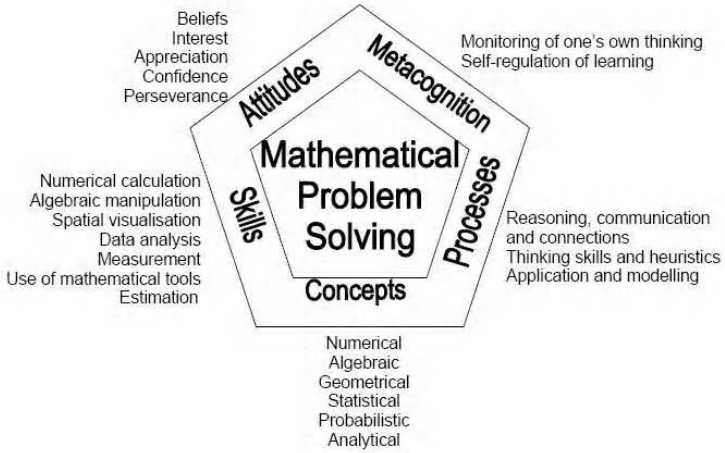
Singapore's Mathematical Problem Solving Framework



Teach Less, Learn More

A call from PM Lee Hsien Loong during his inaugural National Day Rally in 2004 for schools and teachers to:

- *teach less*
- *improve the quality of interaction between teachers and students*
- *equip students with the knowledge, skills and values to prepare them for life*

Educational Achievements

- ✓ High level of achievement in Math & Science – top scorer (TIMMS)
- ✓ Scored well in international test of reading skills (Netherlands-based International Association of Educational Achievement)
- ✓ Performed well in international competitions (Math & Science Olympiads)

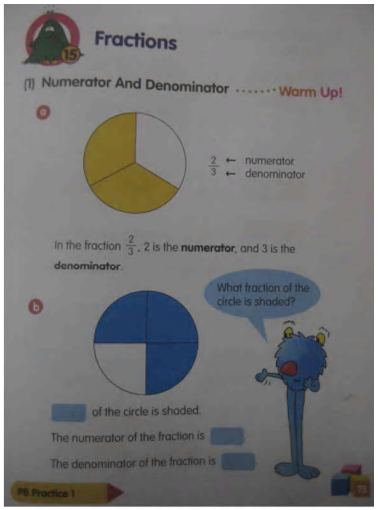
So Why the Need for Curriculum Reforms?

- ✓ Current state may not serve as well in the future
- ✓ 21st century competencies and capacities in response to globalisation and knowledge economies
- ✓ CRPP research on classroom practices – *classroom pedagogy mainly teacher-centred with high levels of students on task behaviour, reliance on textbooks and worksheets*

S'pore Primary Math Syllabus on 'Fractions'

P2	**Fraction of a whole** Include: • interpretation of fraction as part of a whole, • reading and writing fractions, • comparing and ordering - unit fractions, - like fractions. (Denominators of given fractions should not exceed 12.) **Addition and subtraction** Include addition and subtraction of like fractions within one whole. (Denominators of given fractions should not exceed 12.)
P3	**Equivalent fractions** Include: • Recognizing and naming equivalent fractions • Listing the first 5 equivalent fractions of a given fraction • Writing the equivalent fraction of a fraction given the denominator or the numerator • Expressing a fraction in its simplest form • Comparing fractions with respect to half • Comparing and ordering unlike fractions **Addition and subtraction** Include addition and subtraction of two related fractions within one whole. (Denominators of given fractions should not exceed 12.)
P4	**Mixed numbers and improper fractions** **Addition and subtraction** **Fraction of a set of objects** Include interpretation of fraction as part of a set of objects. **Multiplication** Include: • multiplication of a proper/improper fraction and a whole number, • solving up to 2-step word problems involving addition, subtraction and multiplication, • using unitary method to find the whole given a fractional part.
P5	**Concept of fraction as division** **Four operations**
P6	**Four operations**

Equivalent Fractions in the Singapore Textbook



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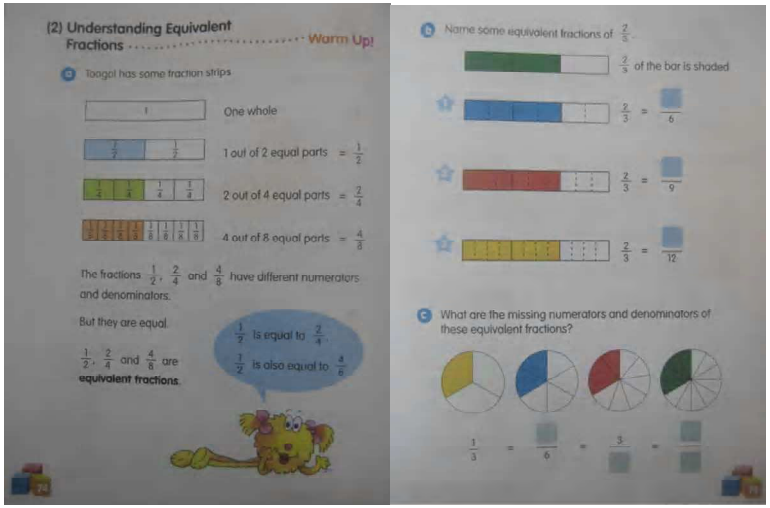

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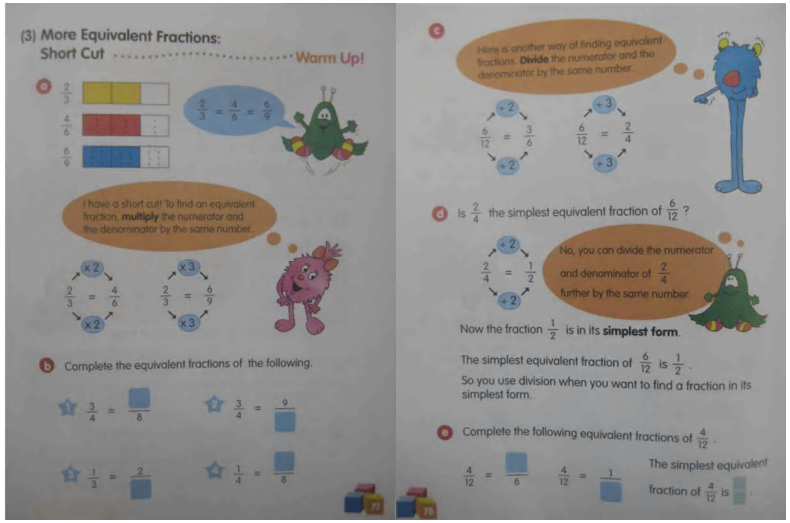
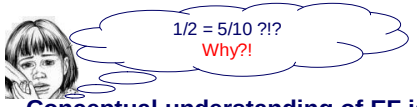


Regular Teaching Practice

- Planning and teaching will normally include
  - Teaching a Mathematical skill
  - Developing Mathematical accuracy
  - Developing speed in Mathematical computation
  - Teach application of a mathematical skill into a different context (usually in the form of word problems).
- Student learning is assessed through feedback from assignments, short quizzes, informal tests, topical tests or mental sums.
- Ultimately, the purpose is to enable students to work efficiently in pencil and paper tests / exams where students will need to maximize their time when answering the questions provided.
- This simply means students are taught "how do I do it?" but not "why am I doing it?".

Why Conceptual Understanding of Equivalent Fractions (EF)?

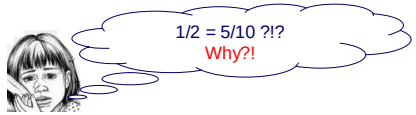


Conceptual understanding of EF is essential for learning of advanced topics in Primary 4, Primary 5 and Primary 6.

Pre-test conducted in 2006

Pupils have difficulties in:	P3	P4	P5
Interpreting fractions as equal parts of a whole.	36.3%		
Ordering unit fractions	2.5%		
Ordering of mixed numbers with unlike and unrelated fractional parts - ascending order		45.8%	
Understanding the term "equivalent"; some thought they referred to factors while others thought they referred to like fractions			65.8%
Subtracting a fraction from a mixed fraction and relied on algorithm knowledge and did not use the visual diagram provided in the test			50.0%

Why Conceptual Understanding of Equivalent Fractions (EF)?



- Abundant studies about children's difficulties in learning fractions (examples: Behr, 1984; Davydov, 1991; Mack, 1990; Kieren, 1993)
- Mathematical rules learnt in Whole Numbers are displaced with a higher level of abstraction in Fractions

As one encounters fractions, the mathematics takes a qualitative leap in sophistication. Suddenly, meanings and models and symbols that worked when adding, subtracting, multiplying, and dividing whole numbers are not as useful. ***Lamon, Susan J. (2006)***

- Hence it is imperative to support the reconstruction of the number system in the child's mind

The Study of Equivalent Fractions @ Cedar Primary

Semester 2 Year 2006

15 Teachers
A Facilitator
A Resource Person
2 Research Lessons

Semester 2 Year 2007

3 Teachers
A Facilitator
4 Critical Friends
3 Resource Persons
2 Research Lessons

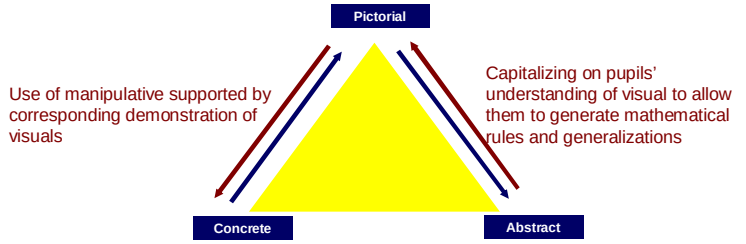
What constitutes Conceptual Understanding of Equivalent Fractions?

- Interpret and articulate the concept of fractions
- Explain understanding using pictorial representations or personal anecdotes
- Generate and explain the short-cut method of equivalent fractions
- Apply the knowledge in problem solving

Concrete-Pictorial-Abstract Model

MOE Syllabus. With Reference to the Textbook "My Pals are Here"

Pictorial representations is the bridge from Concrete to Abstract. Importance of reiteration of concepts by varying the visuals.



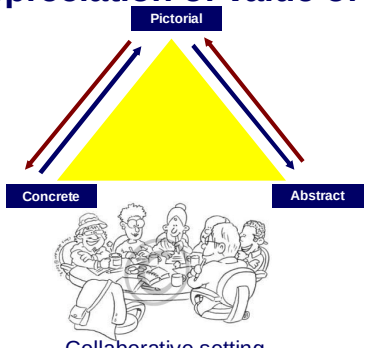
Manipulatives strengthen students' understanding and conception of mathematical concepts (see for example Bruner,1975 & Mason, 1996).

Students become independent of pictorial and concrete in computation of algorithms.

Manipulatives have to be very well thought through

Important for teacher's and students manipulatives to be similar in shape and colour
Circles → Rectangles → Other shapes

Pathway to deepen understanding and increase appreciation of value of CPA Model



Collaborative setting of Lesson Study

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A concerted effort in using CPA model

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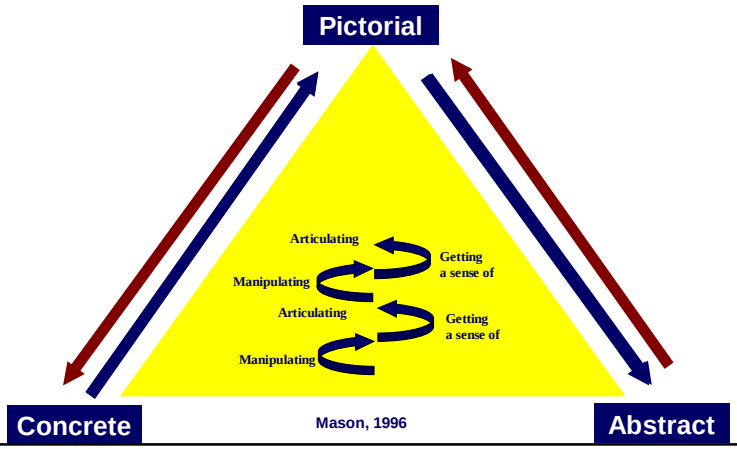
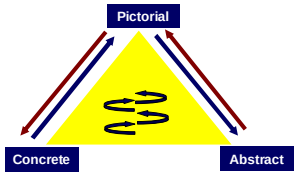
Deeper understanding of the model

Mason's Spiraling Helix Model as a Vehicle in utilisation of CPA model

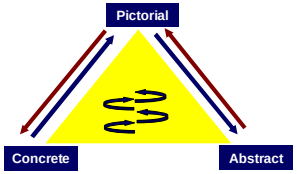


Building a TLLM classroom discourse around the CPA –Learning Helix framework



- Teachers gained a better understanding of how to better support students construction of knowledge on fractions and reconstruction of their knowledge of numbers.
- Consequently this induces the reduction of students dependence on teachers', as they are encouraged to think for themselves and articulate their understanding

Building a TLLM classroom discourse around the CPA –Learning Helix framework



- Challenge: Students rarely get the opportunity to articulate their mathematical thinking in regular lessons.
- Choral reading is a good strategy in building students mathematical vocabulary and confidence
- Piaget (1981) and von Glaserfeld (1990) emphasised that using language advances the child's mental powers. This lends credence to Mason's spiral helix.

4 Research Lessons Across 2 Lesson Study Cycles

Research Lesson 1: 15 August 2006
Mr Sean Teng (Novice Teacher) 1y3m teaching experience
Class Profile: 'Mixed' Ability

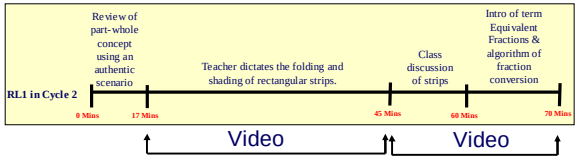
Research Lesson 2: 23 August 2006
Mr Naufal (Novice Teacher) 3m teaching experience
Class Profile: 'Mixed' Ability

Research Lesson 3: 27 August 2007
Mr Naufal (Novice Teacher) 1y3m teaching experience
Class Profile: 'Low' Ability

Research Lesson 4: 21 September 2007
Mrs Elaine Loke (Experienced Teacher) 8y teaching experience
Class Profile: 'Mixed' Ability

4 Research Lessons Across 2 Lesson Study Cycles

How was the concept of Equivalent Fractions addressed?



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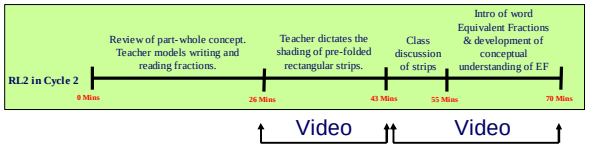

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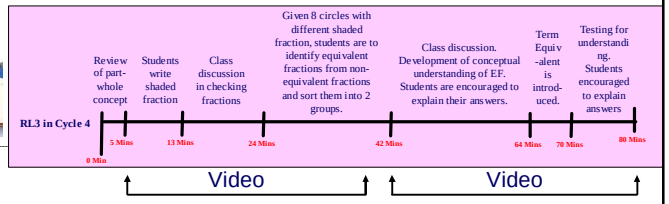

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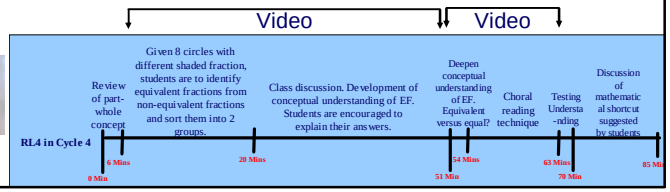
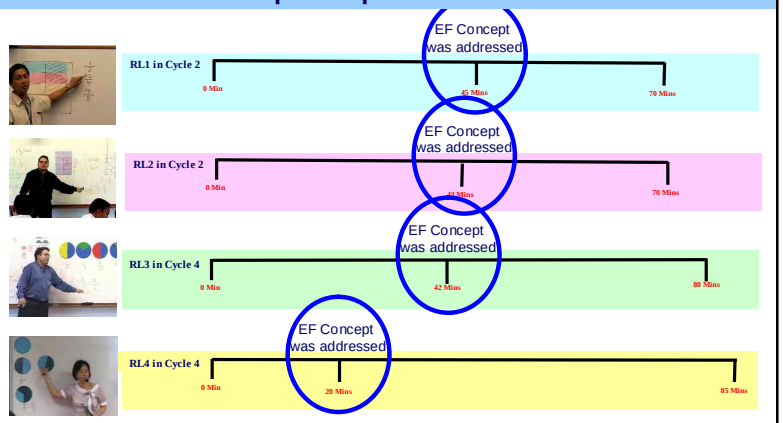


4 Research Lessons Across 2 Lesson Study Cycles

When was the concept of Equivalent Fractions addressed?



Teachers' Learning Points

- Draw on students' prior knowledge of fractions. focused on equivalent fractions
- Teach students to observe, identify and compare the patterns before teaching concept of equivalent fractions
- Represent fractions in concrete terms as well as symbolically
- Capitalize on teachable moments, such as reiteration of the reason why equivalent fractions are represented differently although they are of the same value.

Teachers' Learning Points

- The recapitulation should have been done in the previous lesson and not during the RL.
- There should be more repetition of the teaching points so choral reading is encouraged.
- Most pupils had difficulty articulating as they are not used to the sharing discourse
- The questioning technique is a vital component of the lesson.

Conclusion

- The research lessons have provided an authentic experience for teachers to develop a systematic pedagogical model of using Concrete-Pictorial-Abstract in teaching a difficult math topic.
- The research lessons are not the best lessons to showcase; yet are powerful tools from which continuous improvements may be pursued.
- Given the difficulty of the topic, it is best to extend the lesson study process to include the whole unit of EF, instead of just focusing on the one lesson on conceptual understanding.
- Lesson study an important teacher professional development tool

Acknowledgements

- Mrs Shirley Ho-Woo, Principal of Cedar Primary
- Dr Fang Yan Ping, NIE
- Associate Professor Christine Lee, NIE
- Cedar teachers involved in the 2 EF cycles
- WALS organising committee

How would you teach Equivalent Fractions?