Teaching Arithmetic Mean to Primary Students Using IT: A Case Study

個案研究：利用資訊科技協助小學生學習算術平均數

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Abstract

Primary students generally focus on the mechanical procedures of calculations without really grasping the conceptual knowledge of mathematics. As a result, students often have great difficulties in tackling real-life mathematical problems, as the mere knowing of how to carry out computational procedures does not always empower them to resolve the problems on hand. This deficiency of conceptual knowledge in mathematics and reasoning skills has compelled the author to address the knowledge imbalance in the present paper. Specifically, he wants to outline a wide scope of knowledge about data handling topics in the Primary Mathematics Syllabus and discuss ways of eliminating the deficiency addressed above through the constructivist approach to learning and reinforcement of mathematical knowledge by actively engaging the students in an IT-rich learning environment, as well as in the process of problem solving.

Keywords : Mathematical reasoning skills, constructivist theory, problem solving.

摘要

小學生一般只著眼於運算的機械式過程，而沒有真正掌數學概念。單單懂得運算步驟，並未能協助他們解決實際問題。故此，學生通敘在應付實際生活的數學問題時，通敘都會感到相當吃力。透過本文，作者指出學生對數學概念及推理技巧貧乏的不平衡現象。文中主要描述小學數學課程綱要內有關數據處理的知識，討論以建構主義學習觀解決上述不平衡現象的方法，並讓學生在豐富的資訊科技學習環境下，積極參與學習及解決難題的過程，從而增強數學知識。

Introduction

Much of students' time and effort is devoted to mastering the procedural aspect of calculations without grasping the underlying concepts of mathematical knowledge so that students are unable to tackle real-life mathematical problems, not to mention their incompetence in presenting their findings in a proper and meaningful manner. As such, they lack conceptual knowledge of mathematics and their focus on mathematics learning is quite narrow.

Moore (2000) emphasised that "Procedure and understanding are separate domains". This explains why asking a student to keep on performing mathematical procedures will not help the student to understand the underlying mathematical concepts. Mathematics learning involves both the process and product (Biehler, 1993; NTCM, 1989). The teacher should therefore pay more attention on how to engage students in the learning process; how mathematical knowledge is to be acquired through learning activities organised in an IT-rich learning environment; and how to help the students develop mathematical reasoning and problem-solving skills.

Mathematical Knowledge

Students are expected to learn and reflect on the knowledge by judging its implication, interconnection and utility. Knowledge acquisition is a dynamic process, totally different from learning without assimilation. Students not only need to update, adjust and amend the existing knowledge but also to consolidate knowledge in an integrated manner rather than treating bits and pieces of knowledge as isolated entities. Newly learned knowledge should be fully integrated with prior knowledge and bridged to experience before it becomes usable (Way, 1991). Only when knowledge is consolidated and its structures are organised in an integrated manner can higher-order thinking then be facilitated.

Calculations, graphing, concepts, methods, techniques and communication are some core components of mathematical knowledge. The first two components are the mechanical aspects. According to Hiebert (1986), students must understand the language or symbol representation system of mathematics as well as rules, algorithms or procedures used for mathematical operations prior to accomplishing calculation and graphing tasks. For instance, when students want to calculate a sample mean by using this formula, $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, they must read the mathematical operators and notations and follow syntactic rules for mathematical operations. Prior to mathematical operations, they must keep count of $\bar{x}$ is given by the ratio of $\sum_{i=1}^{n} x_i$ to n. To carry out the first mathematical operation, they need to translate $\sum_{i=1}^{n} x_i$ into our daily language, as in adding all the observed values that are equivalent to $x_1+x_2+\ldots+x_n$. The second mathematical operation they need perform is to divide the sum of all the observed values by the total number of observed values (n).

To have a grasp of conceptual knowledge of sample mean according to Hiebert's definition (1986), students must be aware that n is a positive integer representing the total number of observed values generally larger than one in a set of data, otherwise the value of the sample mean becomes meaningless. Besides, a sample mean is greatly affected by an extremely small or large value that will consequently distort the measure of central tendency for a set of data and is therefore not appropriate for the measure of central tendency (see Figures 2A and 2B). Furthermore, they also need to bear in mind that there is a distinction between a sample mean $\left(\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}\right)$ and a population mean $\left(\mu = \frac{\sum_{i=1}^{N} x_i}{N}\right)$ in a statistical process although these two formulas are computationally equivalent (Li, 2000). Yet the conceptual aspect of mathematical knowledge that students are expected to possess refers to their abilities to render information according to the contextual frame of data; to utilise an appropriate mathematical tool to solve a problem; and to report the findings of their work. Obviously, the mechanical aspects of mathematical knowledge are well taken by our students but it is the conceptual aspects that are deficient (Law, 1996). It is believed that constructivist theory provides principles to guide the design of teaching and learning activities so as to help students understand the conceptual aspects of mathematical knowledge.

Constructivist Theory

Students must have a thorough understanding of the knowledge structure, the fundamental ideas and the inter-relationship among various concepts before knowledge can be fully utilised and retained. Thus, making learning meaningful for students is a key issue in teaching and learning. This will be achieved as long as students are able to relate the new information to prior information and consolidate all information of different areas through personal experiences.

In constructivist theory of learning (Cobb, 1994; Papert, 1980; Scrimshaw, 1993), an emphasis is placed on the student's construction of his or her own understanding. Students are therefore expected to be autonomous, active and independent learners; their teacher provides assistance when necessary. A teacher may pose questions for students to answer by formulating conjectures, making intuitive guesses, exploring patterns, experimenting with data, reasoning about results and so forth. When students work out the solution to a problem through active learning, they will understand the concept more concretely. This is a preferred approach for a positive understanding during a learning session. Students will become more competent at reasoning and have a firm grasp of conceptual knowledge rather than just memorising facts, procedures or formulas.

Nowadays, students are of varying backgrounds, interests and learning pace. To accommodate such heterogeneity, integrated use of IT-based educational materials may be employed to suit individual student's needs; to extend his or her ability to see things hidden or unhidden; to utilise interactive multimedia tools for enhancement of educational process; and to convince him or her of concepts (Barrett, 1994; Li, 2001).

IT-rich Learning Environment

Stat.Net is a bilingual web site (available at http://www.hked-stat.net/common/index.htm) for teaching and learning built by the Education Department of The Hong Kong Special Administrative Region and covers data handling topics in the Primary Mathematics Syllabus (CDC, 2000). It uses interactive components to elaborate and illustrate statistical concepts and principles. And at the same time it also offers students hands-on practice exercises and core features: Topics, Toolbox, On-line Survey Tools, Statistical Data, Games and Exercises, Glossary and Resource Library. Topics give a table of statistical topics such as, Statistical Graphs and Charts, Measures of Central Tendency and Dispersion and so forth. Toolbox offers handy data analysis and statistical graphing tools. On-line Survey Tools enable students to collect their data via the Internet. Statistical Data provide various sets of real-life data with local context, for instance Beach Water Quality (1992-1999), Composition of Domestic Waste

(1986-1998), etc. Games and Exercise offer more examples and discussions of statistical concepts and principles. Glossary provides definitions and explanations of statistical terms. Resource Library provides an on-line teacher's guide, access to useful web sites associated with the teaching and learning of statistics, and the winning projects of the Hong Kong Statistical Project Competition for Secondary School Students (1998-2002).

Seeing Statistics™ (available at http://www.seeingstatistics.com/) is a web book prepared by McClelland (1999) and has sixteen chapters covering the following topics:- 1) Introduction; 2) Data and Comparison; 3) Seeing Data; 4) Describing the Centre; 5) Describing the Spread; 6) Seeing Data, Again; 7) Probability; 8) Normal Distribution; 9) Inference and Confidence; 10) One-Sample Comparisons; 11) Two-Sample Comparisons; 12) Multi-Group Comparisons; 13) Correlation and Regression; 14) Categorical Data; 15) Nonparametrics and Transformations; and 16) Gallery. Chapter 4 is within the scope of the Primary Mathematics Syllabus.

Acquisition of knowledge about mean is indispensable for our daily life and work in this information age but its concept is not thoroughly grasped by most students (Mevarech, 1983; Pollatsek et al., 1981; Watson & Mortiz, 1999). Stat.Net or Seeing Statistics™ offers teachers ample ways of illustrating the concepts of mean with interactive examples that incorporate computer visualisation tools.

Illustration of the concept of mean

To summarise a set of data quantitatively, a teacher introduces the concept of mean. He or she can illustrate how the mean is related to the data spread vividly using computer demonstrations offered by Seeing Statistics™ (see Figure 1). In addition, he or she should facilitate data exploration by asking his or her students to guess the value that can best represent the measure of central tendency. Through this authentic learning activity, the teacher maintains active discussion with his or her students and also demonstrates how close their answers are to the mean. This approach encourages each student to play an active role of constructor in knowledge understanding.

Figure 1. Illustration of the concept of mean

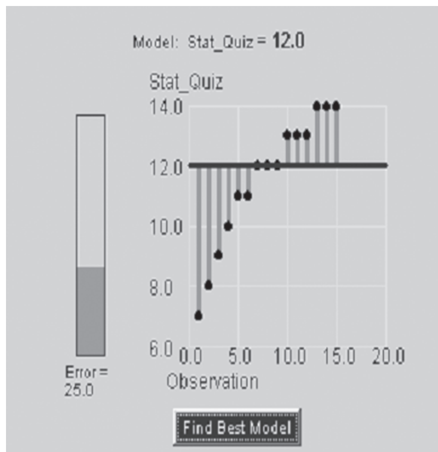


Illustration of the influence of an extreme value

Through use of an interactive exploration tool, teachers interact with an extreme value of a given set of data in illustrating the influence of a single extreme value. Students discern how the mean may be affected by a single observation when moving it from the bottom to the top, as shown in Figures 2A and 2B.

Figure 2A: A single observation close to the set of data

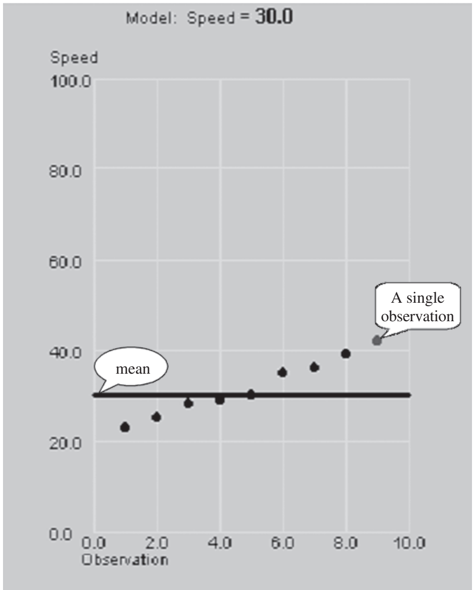
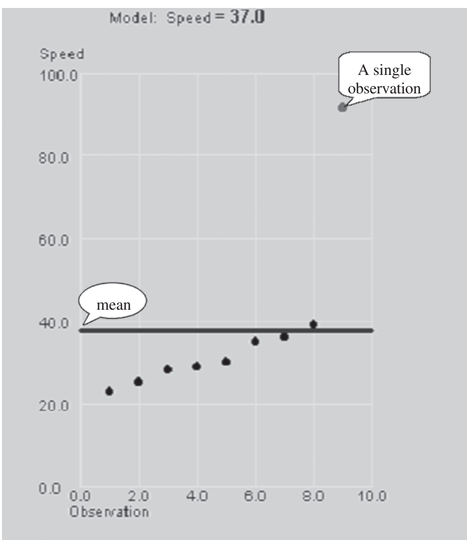


Figure 2B: A single observation remote from the set of data



Implications for a Mathematics Classroom

Applying the constructivist approach to learning in a mathematics class, a teacher should not only provide a clear picture of the inter-relationships among various concepts but also organise the learning experiences in a meaningful context. A clear understanding of mathematical concepts is achieved more easily by students' active involvement in authentic learning activities. Through solving real-life problems and/or projects, they gradually build up knowledge and sharpen their understanding at their own pace. In the case that certain tasks are beyond some students' reach, their teacher should provide an appropriate level of support so as to ensure they are capable of constructing their own meaning of learning. In this way, what is learned is developed into concrete mathematical understanding (Smith, 1998).

Solving textbook mathematical problems hinders the development of students' abilities to explore real world phenomena. Instead, solving practical mathematical problems arising from real-life circumstances may arouse students' interest and stimulate their thinking (Cobb, 1987). This also fulfils some of the learning objectives in the Primary Mathematics Syllabus (CDC, 2000). In addition, this is indeed a pragmatic approach to acquire mathematical knowledge and it offers an opportunity for open-ended exploration by students (Marasinghe et al., 1996). It aims at educating students to become problem solvers by enhancing the learning process; reinforcing mathematical concepts and techniques; improving mathematical thinking and reasoning; studying mathematical phenomena in the real world; etc. (Snee, 1993).

Thus, for example, a teacher can give his or her students a project title, "A study of students' heights in their class". It is too difficult for students to complete the whole project without any prior knowledge of problem solving. The teacher should teach students how to subdivide a project into sub-problems they need to accomplish.

Real-life mathematical problems generally exist in the form of a word problem. Thus, a student needs to read the problem and construct an appropriate problem statement, as these two steps are prerequisites in problem solving. It is imperative that the organised verbal information is employed to translate verbal statements into specific directions for mathematical operations (Francis, 1990). This guides a problem solver in constructing their representation of the problem. Thus, students have to state the problem, and the aims and objectives of the project with regard to how the problem is tackled; what type of data is required; how data will be collected; and what practical significance is attached to the mathematical result.

Effective learning takes place when mathematics is being studied in the context of meaningful problems. When students approach these problems, they have more provision of hands-on practice experience in support of the development of mathematical reasoning (Smith, 1998). To actively participate in the mathematical problem-solving process, students need to collect data in the real world (Cobb, 1987). In the project proposed by the author, students are involved in activities in which they collect data from their own class and search for answers. Collection of raw data is not mandatory, a student can also use the data, compiled and released by the Census and Statistics Department of The Hong Kong Special Administrative Region, to prepare a relevant set of data together with a description of the data context (Li, 2000). However, their teacher should ensure the context of data is relatively simple for primary students such as, temperature, amounts of rainfall and so on.

To eliminate the drudgery of computations, students could use a calculator or handy data analysis tools offered by the Stat.Net to speed up computations so that more time can be spent on exploration of mathematical ideas. Using data to calculate the mean only fulfils the computational needs, the value of mean so obtained may not be practically sound. Students should therefore reason the numerical answer and justify the physical meaning of mean and its usefulness. However, as students may not have this ability of relating and justifying numerical results to a real-life problem, teachers can assist them by posing questions such as,

* Is the mean of your classmates' heights within a reasonable range?
* Is your height above or below the mean?
* Who are the tallest and the shortest classmates?
* What are their heights?
* If their heights are discarded from the data set, what is the new value of mean?
* Comparing the original and new values of mean, is there any difference between these two values?
* Can you summarise how the mean is affected by the heights of the tallest and the shortest classmates?

To attempt the above questions, students need to evaluate the numerical results in the context of classmates' heights rather than treating them as separate entities. Through this learning activity, students can eventually develop reasoning skills (for example, Mathematics Section, 2001; Sobel & Maletsky, 1988) that would equip them to fulfil job requirements in this information age as well as educational objectives in the Primary Mathematics Syllabus (CDC, 2000). This approach can be easily adapted to other problems arising from different circumstances.

The entire problem-solving process is far from complete until all the findings are summarised and presented by means of either verbal or written reports. Writing a brief report is a vital part of mathematical work for presenting analytical results to an audience in a clear and concise manner. In addition, report writing is conducive to the internalisation and conceptualisation of subject material; to encourage creativity; and to enhance the ability of communication (Radke-Sharpe, 1991). Students are therefore required to prepare their own reports.

Conclusion

Mathematics teaching in traditional classrooms focuses much on learning formulas, calculations and graphing by rote rather than supporting conceptual development. The constructivist approach to learning allows each student to play the active role of constructor in knowledge understanding instead of the passive role of knowledge recipient. Teachers therefore need to decide what kinds of activities should engage students during active learning.

A thorough grasp of mathematical knowledge relies on the method by which students undergo concrete experience in their learning activities such as experimentation, exploration and visualisation of data and so on. After participating in hands-on learning activities, students gain a deeper comprehension of mathematical knowledge.

Participating in a project provides an opportunity for performing practical mathematical activity so as to gain a better understanding of the concepts involved. That is, students should use not only previously learned rules in carrying out routine tasks but also genuine knowledge and strategies in tackling a mathematical problem. Instead, the entire problem-solving process involved induces the student to build up mathematical knowledge and develop mathematical reasoning skills that are transferable to other problem contexts.

However, teachers should note that when teaching towards doing a project, the content used as the vehicle should initially be simple and the topic of investigation should be chosen within the capabilities of the class. If a topic or the content is too difficult, students may be disappointed and withdraw from the learning activity.

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